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XVI. *Account of Experiments on the Perception of Colour.*

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

THE experiments which I intend to describe were undertaken in order to render more perfect the quantitative proof of the theory of three primary colours. According to that theory, every sensation of colour in a perfect human eye is distinguished by three, and only three, elementary qualities, so that in mathematical language the quality of a colour may be expressed as a function of three independent variables. There is very little evidence at present for deciding the precise tints of the true primaries. I have ascertained that a certain red is the sensation wanting in colour-blind eyes, but the mathematical theory relates to the number, not to the nature of the primaries. If, with Sir David Brewster, we assume red, blue, and yellow to be the primary colours, this amounts to saying that every conceivable tint may be produced by adding together so much red, so much yellow, and so much blue. This is perhaps the best method of forming a provisional notion of the theory. It is evident that if any colour could be found which could not be accurately defined as so much of each of the three primaries, the theory would fall to the ground. Besides this, the truth of the theory requires that every mathematical consequence of assuming every colour to be the result of mixture of three primaries should also be true.

I have made experiments on upwards of 100 different artificial colours, consisting of the pigments used in the arts, and their mechanical mixtures. These experiments were made primarily to trace the effects of mechanical mixture on various coloured powders; but they also afford evidence of the truth of the theory, that all these various colours can be referred to three primaries. The following experiments relate to the combinations of six well-defined colours only, and I shall describe them the more minutely, as I hope to induce those who have good eyes to subject them to the same trial of skill in distinguishing tints.

The method of performing the experiments is described in the Transactions of the Royal Society of Edinburgh, Vol. XXI. Part 2. The colour-top or teetotum which I used may be had of Mr J. M. Bryson, Edinburgh, or it may be easily extemporized. Any rotatory apparatus which will keep a disc revolving steadily and rapidly in a good light, without noise or disturbance, and can be easily stopped and shifted, will do as well as the contrivance of the spinning-top.

The essential part of the experiment consists in placing several discs of coloured paper of the same size, and slit along a radius, over one another, so that a portion of each is seen, the rest being covered by the other discs. By sliding the discs over each other the proportion of each colour may be varied, and by means of divisions on a circle on which the discs lie, the proportion of each colour may be read off. My circle was divided into 100 parts.

On the top of this set of discs is placed a smaller set of concentric discs, so that when the whole is in motion round the centre, the colour resulting from the mixture of colours of the small discs is seen in the middle of that arising from the larger discs. It is the object of the experimenter to shift the colours till the outer and inner tints appear exactly the same, and then to read off the proportions.

It is easy to deduce from the theory of three primary colours what must be the number of discs exposed at one time, and how much of each colour must appear.

Every colour placed on either circle consists of a certain proportion of each of the primaries, and in order that the outer and inner circles may have precisely the same resultant colour in every respect, there must be the same amount of each of the primary colours in the outer and inner circles. Thus we have as many conditions to fulfil as there are primary colours; and besides these we have two more, because the whole number of divisions in either the outer or the inner circle is 100, so that if there are three primary colours there will be five conditions to fulfil, and this will require five discs to be disposable, and these must be arranged so that three are matched against two, or four against one.

If we take six different colours, we may leave out any one of the six, and so form six different combinations of five colours. It is plain that these six combinations must be equivalent to two equations only, if the theory of three primaries be true.

The method which I have found most convenient for registering the result of an experiment, after an identity of tint has been obtained in the inner and outer circles, is the following:—

Write down the names or symbols of the coloured discs each at the top of a column, and underneath write the number of degrees of that colour observed, calling it + when the colour is in the outer circle, and − when it is in the inner circle; then equate the whole to zero. In this way the account of each colour is kept in a separate column, and the equations obtained are easily combined and reduced, without danger of confounding the colours of which the quantities have been measured. The following experiments were made between the 3rd and 11th of September, 1856, about noon of each day, in a room fronting the north, without curtains or any bright-coloured object near the window. The same combination was never made twice in one day, and no thought was bestowed upon the experiments except at the time of observation. Of course the graduation was never consulted, nor former experiments referred to, till each combination of colours had been fixed by the eye alone; and no reduction was attempted till all the experiments were concluded.

The coloured discs were cut from paper painted of the following colours:—Vermilion, Ultramarine, Emerald-green, Snow-white, Ivory-black, and Pale Chrome-yellow. They are denoted by the letters V, U, G, W, B, Y respectively. These colours were chosen, because each is well distinguished from the rest, so that a small change of its intensity in any combination can be observed. Two discs of each colour were prepared, so that in each combination the colours might occasionally be transposed from the outer circle to the inner.

The first equation was formed by leaving out vermilion. The remaining colours are Ultramarine-blue, Emerald-green, White, Black, and Yellow. We might suppose, that by mixing the blue and yellow in proper proportions, we should get a green of the same hue as the emerald-green, but not so intense, so that in order to match it we should have to mix the green with white to dilute it, and with black to make it darker. But it is not in this way that we have to arrange the colours, for our blue and yellow produce a pinkish tint, and never a green, so that we must add *green* to the combination of blue and yellow, to produce a neutral tint, identical with a mixture of white and black.

Blue, green, and yellow must therefore be combined on the large discs, and stand on one side of the equation, and black and white, on the small discs, must stand on the other side. In order to facilitate calculations, the colours are always put down in the same order; but those belonging to the small discs are marked negative. Thus, instead of writing

$$54U + 14G + 32Y = 32W + 68B,$$

we write

$$+ 54U + 14G - 32W - 68B + 32Y = 0.$$

The sum of all the positive terms of such an equation is 100, being the whole number of divisions in the circle. The sum of the negative terms is also 100.

The second equation consists of all the colours except blue; and in this way we obtain six different combinations of five colours.

Each of these combinations was formed by the unassisted judgment of my eye, on six different occasions, so that there are thirty-six independent observations of equations between five colours.

Table I. gives the actual observations, with their dates.

Table II. gives the result of summing together each group of six equations.

Each equation in Table II. has the sums of its positive and negative coefficients each equal to 600.

Having obtained a number of observations of each combination of colours, we have next to test the consistency of these results, since theoretically two equations are sufficient to determine all the relations among six colours. We must therefore, in the first place, determine the comparative accuracy of the different sets of observations. Table III. gives the averages of the errors of each of the six groups of observations. It appears that the combination IV. is the least accurately observed, and that VI. is the best.

Table IV. gives the averages of the errors in the observation of each colour in the whole series of experiments. This Table was computed in order to detect any tendency to colour-blindness in my own eyes, which might be less accurate in discriminating red and green, than in detecting variations of other colours. It appears, however, that my observations of red and green were more accurate than those of blue or yellow. White is the most easily observed, from the brilliancy of the colour, and black is liable to the greatest mistakes. I would recommend this method of examining a series of experiments as a means of detecting partial colour-blindness, by the different accuracy in observing different colours. The next operation is to combine all the equations according to their values. Each was first multiplied by a coefficient proportional to its accuracy, and to the coefficient of white in that equation. The result of adding all the equations so found is given in equation (W).

Equation (Y) is the result of similar operations with reference to the yellow on each equation.

We have now two equations from which to deduce six new equations, by eliminating each of the six colours in succession. We must first combine the equations, so as to get rid of one of the colours, and then we must divide by the sum of the positive or negative coefficients, so as to reduce the equations to the form of the observed equations. The results of these operations are given in Table V., along with the means of each group of six observations. It will be seen that the differences between the results of calculation from two equations and the six independent observed equations are very small. The errors in red and green are here again somewhat less than in blue and yellow, so that there is certainly no tendency to mistake red and green more than other colours. The average difference between the observed mean value of a colour and the calculated value is ·77 of a degree. The average error of an observation in any group from the mean of that group was ·92. No observation was attempted to be registered nearer than one degree of the top, or $\frac{1}{100}$ of a circle; so that this set of observations agrees with the theory of three primary colours quite as far as the observations can warrant us in our calculations; and I think that the human eye has seldom been subjected to so severe a test of its power of distinguishing colours. My eyes are by no means so accurate in this respect as many eyes I have examined, but a little practice produces great improvement even in inaccurate observers.

I have laid down, according to Newton's method, the relative positions of the five positive colours with which I worked. It will be seen that W lies within the triangle V U G, and Y outside that triangle.

The first combination, Equation I., consisted of blue, yellow, and green, taken in such proportions that their centre of gravity falls at W.

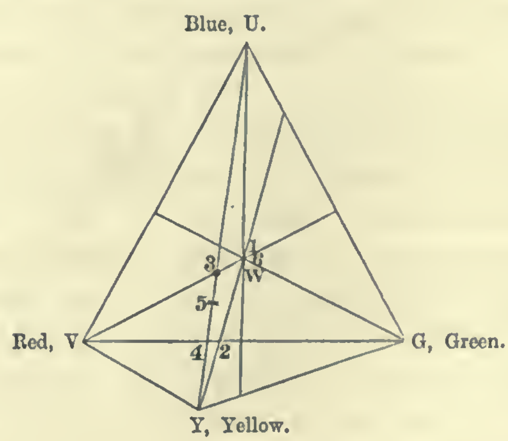
In Equation II. a mixture of red and green, represented in the diagram by the point 2, is seen to be equivalent to a mixture of white and yellow, also represented by 2, which is a pale yellow tint.

Equation III. is between a mixture of blue and yellow and another of white and red. The resulting tint is at the intersection of YU and WV; that is, at the point 3, which represents a pale pink grey.

Equation IV. is between VG and UY, that is, at 4, a dirty yellow.

Equation V. is between a mixture of white, red, and green, and a mixture of blue and yellow at the point 5, a pale dirty yellow.

Equation VI. has W. for its resulting tint.



Of all the resulting tints, that of Equation IV. is the furthest from white; and we find that the observations of this equation are affected with the greatest errors. Hence the importance of reducing the resultant tint to as nearly a neutral colour as possible.

It is hardly necessary for me to observe, that the whole of the numerical results which I have given apply only to the coloured papers which I used, and to them only when illuminated by daylight from the north at mid-day in September, latitude 55°. In the evening, or in winter, or by candlelight, the results are very different. I believe, however, that the results would differ far less if observed by different persons, than if observed under different lights; for the apparatus of vision is wonderfully similar in different eyes, and even in colour-blind eyes the system of perception is not different, but defective.

TABLE I.—The observations arranged in groups.

Equation I.	V=0.	+U.	+G.	−W.	−B.	+Y.
1856, Sept. 3.	0	54	12	34	66	34
4.	0	58	14	31	69	28
5.	0	55	12	32	68	33
6.	0	54	14	32	68	32
8.	0	54	14	32	68	32
9.	0	53	15	32	68	32

Equation II.	−V.	U=0.	−G.	+W.	+B.	+Y.
Sept. 3.	59	0	41	9	71	20
4.	61	0	39	9	68	23
5.	61	0	39	9	67	24
6.	59	0	41	10	66	24
8.	60	0	40	9	69	22
9.	61	0	39	9	68	23

Equation III.	+V.	−U.	G=0.	+W.	+B.	−Y.
Sept. 3.	20	56	0	28	52	44
4.	23	58	0	30	47	42
5.	24	56	0	29	47	44
6.	20	56	0	31	49	44
8.	21	57	0	29	50	43
9.	21	58	0	29	50	42

Equation IV.	−V.	+U.	−G.	W=0.	+B.	+Y.
1856, Sept. 3.	62	15	38	0	53	32
4.	63	17	37	0	46	37
5.	64	16	36	0	50	34
6.	62	19	38	0	46	35
8.	62	19	38	0	47	34
9.	63	17	37	0	49	34

Equation V.	+V.	−U.	+G.	+W.	B=0.	−Y.
Sept. 3.	56	47	28	16	0	53
4.	57	50	25	18	0	50
5.	56	49	24	20	0	51
6.	55	47	27	18	0	53
8.	54	49	26	20	0	51
11.	56	50	27	17	0	50

Equation VI.	+V.	+U.	+G.	−W.	−B.	Y=0.
Sept. 3.	38	27	35	24	76	0
4.	39	27	34	24	76	0
5.	40	26	34	24	76	0
6.	38	28	34	24	76	0
8.	39	28	33	24	76	0
11.	39	27	34	23	77	0

TABLE II.—The sums of the observed equations.

	V.	U.	G.	W.	B.	Y.
Equation I.	0	+328	+ 81	−193	−407	+191
... II.	−361	0	−239	+ 55	+409	+136
... III.	+129	−341	0	+176	+295	−259
... IV.	−376	+103	−224	0	+291	+206
... V.	+334	−292	+157	+109	0	−308
... VI.	+233	+163	+204	−143	−457	0

TABLE III.—The averages of the errors of the several equations from the means expressed in $\frac{1}{100}$ parts of a circle.

Equations.	I.	II.	III.	IV.	V.	VI.
Errors.	·94	·85	1·05	1·17	1·08	·40

TABLE IV.—The averages of the errors of the several colours from the means in $\frac{1}{100}$ parts of a circle.

Colours.	V.	U.	G.	W.	B.	Y.
Errors.	·83	·99	·80	·61	1·15	1·09

Average error on the whole ·92.

The equations from which the reduced results were calculated as follow:—

Equation for (W) = (II) + 2 (III) + (V) − 2 (I) − 4 (VI).

Equation for (Y) = 2 (I) + 2 (II) − 3 (III) + 2 (IV) − 3 (V).

These operations being performed, gave

	V.	U.	G.	W.	B.	Y.	
(W)	+ 701	+ 2282	+ 1060	− 1474	− 3641	+ 1072	= 0.
(Y)	+ 2863	− 2761	+ 1235	+ 1131	+ 299	− 2767	= 0.

From these were obtained the following results by elimination:—

TABLE V.

Equation							
I.	From (W) and (Y)	0	− 54·1	− 13·9	+ 32·0	+ 68·0	− 32·0
	From observation	0	− 54·7	− 13·5	+ 32·1	+ 67·9	− 31·8
II.	From (W) and (Y)	− 59·6	0	− 40·4	+ 10·4	+ 66·0	+ 23·6
	From observation	− 60·2	0	− 39·8	+ 9·2	+ 68·2	+ 22·6
III.	From (W) and (Y)	− 21·7	+ 57·4	0	− 30·2	− 48·1	+ 42·6
	From observation	− 21·5	+ 56·8	0	− 29·3	− 49·2	+ 43·2
IV.	From (W) and (Y)	− 62·4	+ 18·6	− 37·6	0	+ 45·7	+ 35·7
	From observation	− 62·7	+ 17·2	− 37·3	0	+ 48·5	+ 34·3
V.	From (W) and (Y)	+ 55·6	− 49·0	+ 25·2	+ 19·2	0	− 51·0
	From observation	+ 55·7	− 48·7	+ 26·1	+ 18·2	0	− 51·3
VI.	From (W) and (Y)	− 39·7	− 26·6	− 33·7	+ 22·7	+ 77·3	0
	From observation	− 38·8	− 27·2	− 34·0	+ 28·3	+ 76·2	0

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