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VI. *On the Theory of Colours in relation to Colour-Blindness.* A letter to Dr G. Wilson.

DEAR SIR,—As you seemed to think that the results which I have obtained in the theory of colours might be of service to you, I have endeavoured to arrange them for you in a more convenient form than that in which I first obtained them. I must premise, that the first distinct statement of the theory of colour which I adopt, is to be found in *Young's Lectures on Natural Philosophy* (p. 345, Kelland's Edition); and the most philosophical enquiry into it which I have seen is that of Helmholtz, which may be found in the Annals of Philosophy for 1852.

It is well known that a ray of light, from any source, may be divided by means of a prism into a number of rays of different refrangibility, forming a series called a spectrum. The intensity of the light is different at different points of this spectrum; and the law of intensity for different refrangibilities differs according to the nature of the incident light. In Sir John F. W. Herschel's *Treatise on Light*, diagrams will be found, each of which represents completely, by means of a curve, the law of the intensity and refrangibility of a beam of solar light after passing through various coloured media.

I have mentioned this mode of defining and registering a beam of light, because it is the perfect expression of what a beam of light is in itself, considered with respect to all its properties as ascertained by the most refined instruments. When a beam of light falls on the human eye, certain sensations are produced, from which the possessor of that organ judges of the colour and intensity of the light. Now, though every one experiences these sensations, and though they are the foundation of all the phenomena of sight, yet, on account of their absolute simplicity, they are incapable of analysis, and can never become in themselves objects of thought. If we attempt to discover them, we must do so by artificial means; and our reasonings on them must be guided by some theory.

The most general form in which the existing theory can be stated is this,—

There are certain sensations, finite in number, but infinitely variable in degree, which may be excited by the different kinds of light. The compound sensation resulting from all these is the object of consciousness, is a simple act of vision.

It is easy to see that the *number* of these sensations corresponds to what may be called in mathematical language the number of independent variables, of which sensible colour is a function.

This will be readily understood by attending to the following cases:—

1. When objects are illuminated by homogeneous yellow light, the only thing which can be distinguished by the eye is difference of intensity or brightness.

If we take a horizontal line, and colour it black at one end, with increasing degrees of intensity of yellow light towards the other, then every visible object will have a brightness corresponding to some point in this line.

In this case there is nothing to prove the existence of more than one sensation in vision.

In those photographic pictures in which there is only one tint of which the different intensities correspond to the different degrees of illumination of the object, we have another illustration of an optical effect depending on one variable only.

2. Now, suppose that different kinds of light are emanating from different sources, but that each of these sources gives out perfectly homogeneous light, then there will be two things on which the nature of each ray will depend:—(1) its intensity or brightness; (2) its hue, which may be estimated by its position in the spectrum, and measured by its wave length.

If we take a rectangular plane, and illuminate it with the different kinds of homogeneous light, the intensity at any point being proportional to its horizontal distance along the plane, and its wave length being proportional to its height above the foot of the plane, then the plane will display every possible variety of homogeneous light, and will furnish an instance of an optical effect depending on two variables.

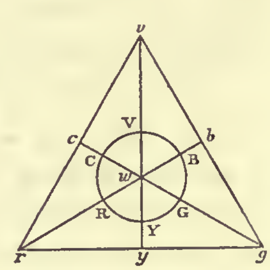
3. Now, let us take the case of nature. We find that colours differ not only in intensity and hue, but also in tint; that is, they are more or less pure. We might arrange the varieties of each colour along a line, which should begin with the homogeneous colour as seen in the spectrum, and pass through all gradations of tint, so as to become continually purer, and terminate in white.

We have, therefore, three elements in our sensation of colour, each of which may vary independently. For distinctness sake I have spoken of intensity, hue, and tint; but if any other three independent qualities had been chosen, the one set might have been expressed in terms of the other, and the results identified.

The theory which I adopt assumes the existence of three elementary sensations, by the combination of which all the actual sensations of colour are produced. It will be shewn that it is not necessary to specify any given colours as typical of these sensations. Young has called them red, green, and violet; but any other three colours might have been chosen, provided that white resulted from their combination in proper proportions.

Before going farther I would observe, that the important part of the theory is not that three elements enter into our sensation of colour, but that there are only three. Optically, there are as many elements in the composition of a ray of light as there are different kinds of light in its spectrum; and, therefore, strictly speaking, its nature depends on an infinite number of independent variables.

I now go on to the geometrical form into which the theory may be thrown. Let it be granted that the three pure sensations correspond to the colours red, green, and violet, and that we can estimate the intensity of each of these sensations numerically.



Let v, r, g be the angular points of a triangle, and conceive the three sensations as having their positions at these points. If we find the numerical measure of the red, green, and violet parts of the sensation of a given colour, and then place weights proportional to these parts at r, g, and v, and find the centre of gravity of the three weights by the ordinary process, that point will be the position of the given colour, and the numerical measure of its intensity will be the sum of the three primitive sensations.

In this way, every possible colour may have its position and intensity ascertained; and it is easy to see that when two compound colours are combined, their centre of gravity is the position of the new colour.

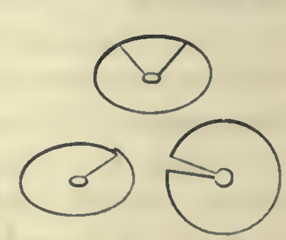
The idea of this geometrical method of investigating colours is to be found in Newton's *Opticks* (Book I., Part 2, Prop. 6), but I am not aware that it has been ever employed in practice, except in the reduction of the experiments which I have just made. The accuracy of the method depends entirely on the truth of the theory of three sensations, and therefore its success is a testimony in favour of that theory.

Every possible colour must be included within the triangle rgv. White will be found at some point, w, within the triangle. If lines be drawn through w to any point, the colour at that point will vary in hue according to the angular position of the line drawn to w, and the purity of the tint will depend on the length of that line.

Though the homogeneous rays of the prismatic spectrum are absolutely pure in themselves, yet they do not give rise to the "pure sensations" of which we are speaking. Every ray of the spectrum gives rise to all three sensations, though in different proportions; hence the position of the colours of the spectrum is not at the boundary of the triangle, but in some curve $C\ R\ Y\ G\ B\ V$ considerably within the triangle. The nature of this curve is not yet determined, but may form the subject of a future investigation *.

All natural colours must be within this curve, and all ordinary pigments do in fact lie very much within it. The experiments on the colours of the spectrum which I have made are not brought to the same degree of accuracy as those on coloured papers. I therefore proceed at once to describe the mode of making those experiments which I have found most simple and convenient.

The coloured paper is cut into the form of discs, each with a small hole in the centre, and divided along a radius, so as to admit of several of them being placed on the same axis, so that part of each is exposed. By slipping one disc over another, we can expose any given portion of each colour. These discs are placed on a little top or teetotum, consisting of a flat disc of tin-plate and a vertical axis of ivory. This axis passes through the centre of the discs, and the quantity of each colour exposed is measured by a graduation on the rim of the disc, which is divided into 100 parts.



* [See the author's Memoir in the *Philosophical Transactions*, 1860, on the Theory of Compound Colours, and on the relations of the Colours of the Spectrum.]

By spinning the top, each colour is presented to the eye for a time proportional to the angle of the sector exposed, and I have found by independent experiments, that the colour produced by fast spinning is identical with that produced by causing the light of the different colours to fall on the retina at once.

By properly arranging the discs, any given colour may be imitated and afterwards registered by the graduation on the rim of the top. The principal use of the top is to obtain colour-equations. These are got by producing, by two different combinations of colours, the same mixed tint. For this purpose there is another set of discs, half the diameter of the others, which lie above them, and by which the second combination of colours is formed.

The two combinations being close together, may be accurately compared, and when they are made sensibly identical, the proportions of the different colours in each is registered, and the results equated.

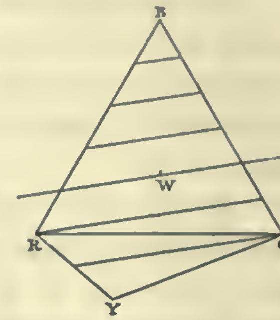
These equations in the case of ordinary vision, are always between four colours, not including black.

From them, by a very simple rule, the different colours and compounds have their places assigned on the triangle of colours. The rule for finding the position is this:—Assume any three points as the positions of your three standard colours, whatever they are; then form an equation between the three standard colours, the given colour and black, by arranging these colours on the inner and outer circles so as to produce an identity when spun. Bring the given colour to the left-hand side of the equation, and the three standard colours to the right hand, leaving out black, then the position of the given colour is the centre of gravity of three masses, whose weights are as the number of degrees of each of the standard colours, taken positive or negative, as the case may be.

In this way the triangle of colours may be constructed by scale and compass from experiments on ordinary vision. I now proceed to state the results of experiments on Colour-Blind vision.

If we find two combinations of colours which appear identical to a Colour-Blind person, and mark their positions on the triangle of colours, then the straight line passing through these points will pass through all points corresponding to other colours, which, to such a person, appear identical with the first two.

We may in the same way find other lines passing through the series of colours which appear alike to the Colour-Blind. All these lines either pass through one point or are parallel, according to the standard colours which we have assumed, and the other arbitrary assumptions we may have made. Knowing this law of Colour-Blind vision, we may predict any number of equations which will be true for eyes having this defect.



The mathematical expression of the difference between Colour-Blind and ordinary vision is, that colour to the former is a function of two independent variables, but to an ordinary eye, of three; and that the relation of the two kinds of vision is not arbitrary, but indicates the absence of a determinate sensation, depending perhaps upon some undiscovered structure or organic arrangement, which forms one-third of the apparatus by which we receive sensations of colour.

Suppose the absent structure to be that which is brought most into play when red light falls on our eyes, then to the Colour-Blind red light will be visible only so far as it affects the other two sensations, say of blue and green. It will, therefore, appear to them much less bright than to us, and will excite a sensation not distinguishable from that of a bluish-green light.

I cannot at present recover the results of all my experiments; but I recollect that the neutral colours for a Colour-Blind person may be produced by combining 6 degrees of ultramarine with 94 of vermilion, or 60 of emerald-green with 40 of ultramarine. The first of these, I suppose to represent to our eyes the kind of red which belongs to the red sensation. It excites the other two sensations, and is, therefore, visible to the Colour-Blind, but it appears very dark to them and of no definite colour. I therefore suspect that one of the three sensations in perfect vision will be found to correspond to a red of the same hue, but of much greater purity of tint. Of the nature of the other two, I can say nothing definite, except that one must correspond to a blue, and the other to a green, verging to yellow.

I hope that what I have written may help you in any way in your experiments. I have put down many things simply to indicate a way of thinking about colours which belongs to this theory of triple sensation. We are indebted to Newton for the original design; to Young for the suggestion of the means of working it out; to Prof. Forbes* for a scientific history of its application

* *Phil. Mag.* 1848.

to practice; to Helmholtz for a rigorous examination of the facts on which it rests; and to Prof. Grassman (in the *Phil. Mag.* for 1852), for an admirable theoretical exposition of the subject. The colours given in Hay's *Nomenclature of Colours* are illustrations of a similar theory applied to mixtures of pigments, but the results are often different from those in which the colours are combined by the eye alone. I hope soon to have results with pigments compared with those given by the prismatic spectrum, and then, perhaps, some more definite results may be obtained. Yours truly,

J. C. MAXWELL.

EDINBURGH, 4th *Jan.* 1855.